

Just Watch that Pumpkin Fly!

Investigating the Vertex of a Quadratic Function

11.5

LEARNING GOALS

In this lesson, you will:

- Interpret parts of a quadratic function in terms of a problem situation.
- Use a calculator to determine the x -intercept(s), y -intercept, and absolute maximum or minimum of a quadratic function.
- Solve a quadratic function graphically.
- Determine the vertex of a quadratic function.
- Use symmetric points to determine the location of the vertex of a parabola.
- Use the vertex to determine symmetric points on a parabola.

KEY TERMS

- vertex
- axis of symmetry

Every year the county of Sussex, Delaware, holds a competition called the Punkin' Chunkin' World Championships, which is a pumpkin-throwing competition. Participants build machines that hurl pumpkins great distances. The winner is the person whose machine hurls the pumpkin the farthest.

There are different divisions based on the type of machine used.

- The Air Cannon Division includes machines that use compressed air to fire pumpkins.
- In the Catapult Division, catapults are composed of cords, springs, rubber, weights, or other mechanisms that create and store energy.
- The Centrifugal Division includes machines that have devices that spin at least one revolution before firing pumpkins.
- There is also the Trebuchet Division. Trebuchets are machines that have swinging or fixed counterweights that can fling pumpkins up and through the air.

What do you think the world-record distance is for hurling a pumpkin at the Punkin' Chunkin' World Championships? Which machine do you think was used?

PROBLEM 1 Punkin Chunkin by Catapult!

You can model the motion of a pumpkin released from a catapult using a vertical motion model. Remember, a vertical motion model is a quadratic equation that models the height of an object at a given time. The equation is of the form

$$y = -16t^2 + v_0t + h_0,$$

where y represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial vertical velocity (speed) of the object in feet per second, and h_0 represents the initial height of the object in feet.

1. Why do you think it makes sense that this situation is modeled by a quadratic function?



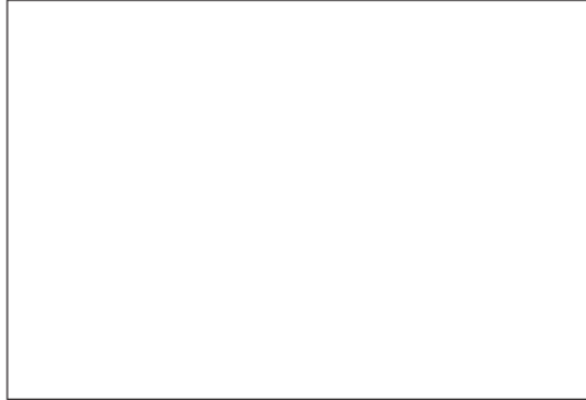
Suppose that a catapult hurls a pumpkin from a height of 68 feet at an initial vertical velocity of 128 feet per second.

2. Write a function for the height of the pumpkin $h(t)$ in terms of time t .

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3. Does the function you wrote have an absolute minimum or an absolute maximum?
How can you tell from the form of the function?

4. Graph the function on a graphing calculator using the bounds $[-1, 9] \times [0, 500]$. Sketch your graph and label the axes.



5. Use a graphing calculator to determine the zeros of the function. Then explain what each means in terms of the problem situation. Do each make sense in terms of this problem situation?

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6. Determine the y -intercept and interpret its meaning in terms of this problem situation.



7. Use a graphing calculator to determine the absolute minimum or maximum. Then explain what it means in terms of this problem situation.

PROBLEM 2 The Vertex of a Parabola and Symmetry

The **vertex** of a parabola is the lowest or highest point on the curve. The **axis of symmetry** of a parabola is the vertical line that passes through the vertex and divides the parabola into two mirror images.

You have been calling the lowest or highest point on a parabola an absolute minimum or maximum. Now you know that this point has a special name—the vertex.



Because the axis of symmetry always divides the parabola into two mirror images, you can say that a parabola is symmetric.



1. Because a parabola is symmetric, over which line would you fold it to show its symmetry? What is the equation of that line?

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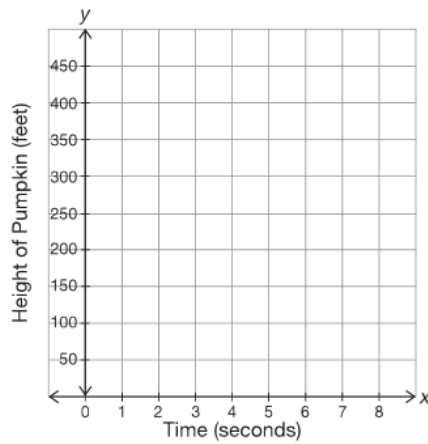
Remember the Punkin Chunkin scenario in Problem 1, in which you wrote the function $h(t) = -16t^2 + 128t + 68$ for the height of the pumpkin in terms of time.

2. Identify the coordinates of the vertex of the graph and the equation for the axis of symmetry.



3. Use a graphing calculator to answer each question.
 - a. When does the pumpkin reach a height of 128 feet?
 - b. When does the pumpkin reach a height of 180 feet?
 - c. When does the pumpkin reach a height of 308 feet?

4. Use the information from Questions 2 and 3 to construct a graph.
 - a. Plot and label the vertex. Then draw and label the axis of symmetry.
 - b. Plot and label the points that correspond to the answers from Question 3.
 - c. Plot and label the points symmetric to the points from part (b).
 - d. Plot and label the zeros.



5. Analyze the symmetric points.
 - a. What do you notice about the y -coordinates?
 - b. What do you notice about each point's horizontal distance from the axis of symmetry?



6. How does the x -coordinate of each symmetric point compare to the x -coordinate of the vertex.

Talk the Talk

Use the given information to answer each question. Do not use a graphing calculator. Show your work.

1. Determine the axis of symmetry of each parabola.
 - a. The x -intercepts of the parabola are $(1, 0)$ and $(5, 0)$.

Sketch a graph by hand if you need a model.



- b. The x -intercepts of the parabola are $(-3.5, 0)$ and $(4.1, 0)$.

- c. Two symmetric points on the parabola are $(-7, 2)$ and $(0, 2)$.

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- d. Describe how to determine the axis of symmetry given the x -intercepts of a parabola.

2. Determine the location of the vertex of each parabola.

- a. The function $f(x) = x^2 + 4x + 3$ has the axis of symmetry $x = -2$.

b. The equation of the parabola is $y = x^2 - 4$, and the x -intercepts are $(-2, 0)$ and $(2, 0)$.

c. The function $f(x) = x^2 + 6x - 5$ has two symmetric points $(-1, -12)$ and $(7, -12)$.

d. Describe how to determine the vertex of a parabola given the equation and the axis of symmetry.

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3. Determine another point on each parabola.

- a. The axis of symmetry is $x = 2$.
A point on the parabola is $(0, 5)$.
Another point on the parabola:

- b. The vertex is $(0.5, 9)$.
An x -intercept is $(-2.5, 0)$.
Another point on the parabola:

- c. The vertex is $(-2, -8)$.
A point on the parabola is $(-1, -7)$.
Another point on the parabola:

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- d. Describe how to determine another point on a parabola if you are given one point and the axis of symmetry.



Be prepared to share your solutions and methods.